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RAPIDLY CONVERGENT ALGORITHMS FOR NONSMOOTH  
OPTIMIZATION(U) WASHINGTON STATE UNIV PULLMAN  
R MIFFLIN 14 JUL 85 AFOSR-TR-85-0713 AFOSR-83-0210

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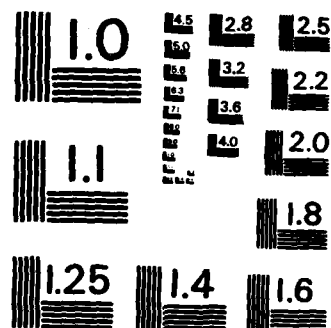
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) <p>► The research supported under this grant has led to new developments for solving nonlinear optimization problems involving functions that are not everywhere differentiable and/or are implicitly defined, such as those that arise from dual formulations of optimization models.</p> <p>A rapidly convergent, both in the theoretical and the practical sense, algorithm has been developed for the single variable case where generalized derivatives</p>		

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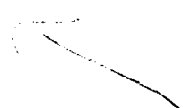
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are available. It is being extended to the case where only function values are known.

Some of the single variable results, including the concept of better than linear convergence, have been extended to the multivariable case. In order to solve efficiently the particular quadratic programming subproblems generated by the n-variable method a specialized QP algorithm has been developed.

*Additional keywords: nondifferential programming*

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The research conducted under Grant Number AFOSR-83-0210 during the period 15 July 1984 to 14 July 1985 is partially reported on in [1], [7], [8], and [9].

The goal of this project is to develop methods to solve efficiently constrained minimization problems that have problem functions that are not everywhere differentiable. Such difficult problems occur in practice when, decomposition, nested dissection, relaxation, duality and/or exact  $L_1$  penalty techniques are applied to large or complicated nonlinear programming problems in order to convert them to a sequence of smaller or less complex problems. Being able to use these techniques that often give implicitly defined problem functions gives a user flexibility in modeling a problem for solution and the ability to exploit parallel processing in computation.

For the case of minimization of a function of a single variable where generalized derivatives are known this research has produced a rapidly convergent algorithm [6] and a corresponding easy-to-use FORTRAN subroutine, called PQ1 [8]. PQ1 was used to solve both smooth [8] and nonsmooth [9] versions of a practical single resource allocation problem [3] having five bounded decision variables via a dual (min-max) technique. This application employed PQ1 in a nested manner, i.e. a single variable outer problem was solved where each function evaluation involved solving a five variable inner Lagrangian problem. The inner problem separated into five independent single variable problems that could have been solved in parallel. The original five variable primal problem is smooth. To make a nonsmooth version the five single variable functions in the objective were approximated with piecewise affine functions. Both versions were solved with about the same amount of computational effort which was about half the effort required in the best run of some popular nonlinear programming codes applied to the original primal problem [3]. These results were reported at the NATO-Advanced

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Study Institute on Computational Mathematical Programming held at Bad Windsheim in July 1984 and at the IIASA Workshop on Nondifferentiable Optimization held at Sopron in September 1984. Also, PQ1 performed well for a referee of [8] and for C. Lemarechal who used it as a line search subroutine in his first order nonsmooth optimization subroutine.

With J-J. Strodiot the principal investigator is preparing papers on a related algorithm for single variable minimization which uses only function values. The method uses function values at five points and employs quadratic and polyhedral approximation together with a theoretical and numerical safeguard. The basic method without the safeguard exhibits a type of better than linear convergence [7] for certain nonsmooth functions. The safeguard keeps apart iterates used in function approximation, guarantees convergence to a stationary point for lower semicontinuous functions [10] and via its order 2 form preserves the better than linear convergence of the basic method for certain continuous piecewise  $C^2$  functions. A preliminary presentation on this work was made at the Cambridge Optimization Symposium held in England in March 1985.

The principal investigator and his graduate student N. Gupta are working on a safeguarded piecewise quadratic approximation technique for the multi-variable case. The attempt here is to obtain a quasi-Newton method for n-variable nonsmooth problems where several "Hessian" matrices are stored, updated and used to generate data for quadratic programming search direction finding subproblems. Some preliminary theoretical results have been obtained showing convergence to stationary points for semismooth functions [4] and giving limiting conditions on the matrices which imply better than linear convergence for certain functions with an underlying piecewise  $C^2$  structure. These results also apply to constrained problems having a strongly convex constraint function and a constraint set with a strict interior via an n-variable

generalization of the scale-free automatic penalty technique in [6]. A preliminary code for the unconstrained case has performed well enough on two test problems, including a ten variable one, to encourage following this line of research.

The quadratic programming subproblems are currently solved with the general purpose code QPSOL [2], but some efficiency can be gained by employing a code based on a specialized algorithm in the thesis [1] of the principal investigator's former graduate student A. Al-Saket. Her work is an extension of the numerically stable constrained least squares algorithm in [5] to problems having more general quadratic objectives.

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